Supersonic Vortex Gerdien Arc with Magnetic Thermal Insulation

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Temperatures up to $\sim 5 \times 10^4$ °K have been obtained with water vortex Gerdien arcs, and temperatures of $\sim 10^5$ °K have been reached in hydrogen plasma arcs with magnetic thermal insulation through an externally applied strong magnetic field.

It is suggested that a further increase in arc temperatures up to 10^6 °K can conceivably be attained by a combination of both techniques, using a Gerdien arc with a supersonic hydrogen gas vortex.

1. Introduction

There is a growing interest to study the spectra of highly ionized atoms, with the aim to identify possible candidates for a soft X-ray laser, aside from other important research goals. The conceptually most simple way to produce a sufficient number of highly ionized atoms is by placing them in a high temperature plasma. Of all the methods to produce a steady state high temperature plasma, the electric discharge is probably the most simple. However, because electric arcs are heated resistively, with the electric conductivity increasing with the 3/2 power of the plasma temperature, the heating of arcs becomes increasingly difficult at higher temperatures. Because of this limitation it is probably not possible to reach arc temperatures in excess of 10^6 °K. Even though this temperature falls short by more than two orders of magnitude from what is needed for thermonuclear fusion, the attainment of a million degree under the steady state condition of an arc, would be most interesting and may have many unforeseen application. It could, for example, serve as a source to pump soft X-ray lasers. Projectiles driven by the arc plasma may reach a velocity sufficient for impact fusion, to mention one other application having a great potential. The arc could also serve as an intense radiation source, reaching from the far UV to soft X-rays. It therefore could serve as a cheap competitor to synchrotron radiation sources, and which depend on expensive electron accelerators.

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In the presented theoretical study we use a greatly simplified arc model by making the assumption, that the arc can be approximated by a plasma cylinder of constant temperature. This assumption requires that the radius of the arc is small against its length. A radial temperature gradient is, of course, still needed to account for the radial heat conduction losses. The arc temperature is therefore in reality its radial average. Near the middle of the arc column the chosen model should make reasonable predictions.

2. Gerdien Arcs

It was the concept of the Gerdien arc [1], with which arc temperatures about one order of magnitude larger were reached, than what was until then thought to be attainable with arcs. Prior to the discovery of the Gerdien arc concept, the stable confinement of an arc in a rotating gas column was proposed as early as 1909 by Schönherr [2]. Because an arc has a lower gas density than its much cooler environment, it is subject to a buoyancy force. Therefore, if an arc is put into a rotating cylindrical gas column, it will be subject to a buoyancy force with a direction opposite to the centrifugal force, and as a result will be stably confined along the axis of rotation.

The decisive step, which in the Gerdien arc principle goes beyond the old idea of Schönherr, is the replacement of the rotating gas column by a liquid vortex. A vortex is hydrodynamically stable by virtue of the centrifugal forces. Because the vortex in a liquid has a hollow core, an arc burning through the core will be subject to a much larger buoyancy force than in a rotating gas column. As a result, the arc plasma is stably confined inside the core of the vortex. Further-

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more, since the radius of the vortex core can be chosen within large limits by a variation of the angular momentum of the liquid injected at the periphery of the cylindrical arc chamber, the vortex core radius and hence arc radius can be made very small.

The importance of having control over the arc radius becomes apparent from Ohm's law

$$\boldsymbol{j} = \sigma \boldsymbol{E}, \tag{1}$$

where $\boldsymbol{j}$ is the electric current density, $\boldsymbol{E}$ the applied electric field and σ the plasma conductivity given by (T absolute temperature):

$$\sigma = a\,T^{3/2}, \quad a = \text{const.} \tag{2}$$

We adopt a simple arc model by assuming that the arc consists of a plasma column of radius r and average temperature T. Applied to this model Ohm's law is

$$T^{3/2} = I/\pi r^2 a E. \tag{3}$$

Expressing I in Ampere and E in Volt/cm, one has

$$T^{3/2} = 6.4 \times 10^4\, I/\pi r^2 E. \tag{4}$$

From Ohm's law it therefore follows that for a constant electric arc resistance per unit arc length E/I, the arc temperature rises as $r^{-4/3}$ with decreasing arc radius. Typically, arcs may have a temperature of a few 10^3 °K and a radius of a cm. Since it is not difficult to make a water vortex with a core diameter of the order 0.1 cm, it is no wonder that water vortex Gerdien arcs have reached temperatures up to $\sim 5 \times 10^4$ °K, that is about one order of magnitude larger than it was possible before [3–6].

To reach substantially higher temperatures with this same technique will be very difficult, considering the Elenbaas-Heller equation, after Ohm's law the 2nd most important equation of arc physics. The Elenbaas-Heller equation describes the balance of the electric energy input with the radial heat conduction losses, and is given by

$$\boldsymbol{j} \cdot \boldsymbol{E} = -\operatorname{div}(\varkappa \operatorname{grad} T) \tag{5}$$

where

$$\varkappa = b\,T^{5/2}, \quad b = \text{const} \tag{6}$$

is a temperature dependent heat conduction coefficient. Integration of (5) over a cylindrical arc column results in

$$I E = -2\pi r \varkappa\, \mathrm{d}T/\mathrm{d}r \tag{7}$$

A second integral of (7) is

$$I E = \frac{2\pi}{\ln(r_a/r)} \int_0^T \varkappa(T)\,\mathrm{d}T, \tag{8}$$

where r_a is the inner radius of the cylindrical discharge chamber. Inserting the expression for $\varkappa(T)$ given by (6) into (8) one obtains

$$I E = (4\pi b/7 \ln(r_a/r))\, T^{7/2}. \tag{9}$$

The heat conduction losses therefore rise with the 7/2 power of the temperature. Inserting numbers valid for hydrogen one has (I in Ampere and E in Volt/cm):

$$I E = 3.5 \times 10^{-10}\, T^{7/2}/\ln(r_a/r) \ \ [\mathrm{W/cm}]. \tag{10}$$

The first difficulty which stood in the way to reach high arc temperatures, and which was that arcs normally are not confined into a narrow channel where the resistive heating is high, could be solved with the Gerdien arc principle. But now (10) shows that there is a second, by no means less important difficulty in reaching high arc temperatures, and which is caused by the radial heat conduction losses. These losses imply, that a two-fold increase in the arc temperature for example, would require a $2^{7/2} \sim 11$ times larger input power. And a tenfold increase in the arc temperature would even mean a 3000 times larger input power. For Gerdien arcs which at a temperature of 5×10^4 °K operate with an input power maximal $\sim 10^6$ Watt, a tenfold increase in the arc temperature to 5×10^5 °K would therefore mean an input power of at least 3×10^9 Watt. If the radiation losses are taken into account the required input power would be even larger. For optically thick arcs the radiation losses at $\sim 10^6$ °K would be disastrous.

3. Magnetic Thermally Insulated Arcs

The only way by which the radiation losses can be substantially reduced is to use a hydrogen (and to a lesser degree helium) plasma. For hydrogen the ionization temperature is less than 10^5 °K. Above this temperature a hydrogen plasma, with a density which is typically for an arc, becomes optically transparent, and its radiation loss increases thereafter only slowly with the 1/2 power of the temperature.

A hydrogen plasma however, does not solve the problem of the radial heat conduction losses and which rise with the 7/2 power of the temperature. A solution to this problem was proposed by Alfvén et al. [7], who showed that the radial heat conduction losses can be substantially reduced if the arc is surrounded by a strong magnetic field. For this purpose the direction of the field must be perpendicular to the direction of the temperature gradient. One may achieve this by simply putting an arc inside a solenoid, with the arc centered on the axis of the solenoid.

Implementing this idea, hydrogen arc temperatures of $\sim 10^5$ °K were obtained by Mahn et al [8]. In this experiment it was also observed that the arc column gets smaller if a magnetic field is applied. The explanation for this behavior was later given by Grassmann et al. [9], as being caused by the thermomagnetic Nernst effect.

The heat conduction coefficient across a strong magnetic field has been computed by Braginskii [10], and is

$$\varkappa = \frac{3}{4\sqrt{\pi}} \frac{k(kT)^{5/2}}{\sqrt{m_i}\, e^4 \lambda} \frac{2.645 + 2(\omega_i \tau_i)^2}{0.677 + 2.70(\omega_i \tau_i)^2 + (\omega_i \tau_i)^4}, \tag{11}$$

where

$$\omega_i \tau_i = \frac{eH}{m_i c} \frac{3\sqrt{m_i}(kT)^{3/2}}{4\sqrt{\pi}\, e^4 n_i \lambda} \tag{12}$$

is the product of the gyrofrequency and collision time of the arc ions. λ is the Coulomb logarithm. For a constant arc pressure the ion density varies as T^{-1}, and $\omega_i \tau_i$ is then proportional to $T^{5/2}$. According to (11) $\varkappa$ therefore decreases for large values of T in proportion to $T^{-5/2}$ (instead of increasing in proportion to $T^{5/2}$ as in the absence of a magnetic field).

As before, the expression for $\varkappa(T)$ must be inserted in the Elenbaas Heller equation (8). Since $\varkappa(T)$ falls off rapidly with rising T, the integral $\int_0^T \varkappa(T)\,\mathrm{d}T$ can be approximated by $\int_0^\infty \varkappa(T)\,\mathrm{d}T$. With this approximation one obtains for a hydrogen plasma [7]:

$$IE = bKT_1^{7/2}, \tag{13}$$

where

$$T_1 = 4.0 \times 10^4 (P/H)^{2/5}. \tag{14}$$

T_1 is approximately the temperature of the maximum ion conductivity where $\omega_i \tau_i \sim 1$. $p = nkT$ is the arc plasma pressure, and $K = 0.53 \times 10^{-14}$ W/cm. Finally

$$b = \frac{2\pi}{\ln(r_a/r)} \int_0^\infty \frac{x^{5/2}(1 + 0.756\,x^5)}{1 + 3.99\,x^5 + 1.48\,x^{10}}\,\mathrm{d}x \simeq 6{,}9/\ln(r_a/r). \tag{15}$$

One therefore has

$$IE = \frac{4.6 \times 10^2}{\ln(r_a/r)} \left(\frac{P}{H}\right)^{1.4} \text{[W/cm]}. \tag{16}$$

Comparing this result with (10) we note that the power here does not depend on the arc temperature, but rather decreases inversely proportional to the 7/5 power of the strength of the applied magnetic field.

Ohm's law in the form of (4) remains, of course, unchanged, and the conclusion drawn from it that for constant arc resistance the temperature rises as $r^{-4/3}$ still holds. But now we cannot raise the arc temperature by reducing the arc radius through external manipulation, as it was possible with the Gerdien arc, but this disadvantage is more or less compensated by the greatly reduced radial heat conduction losses, resulting in the high arc temperatures achieved, and which are of the same order of magnitude as in Gerdien arcs.

More important is another problem. It has to do with the end losses and prevents the attainment of temperatures well in excess of 10^5 °K. Because only if the ratio of the arc radius to its length is sufficiently small, can one neglect the end losses and only then does (16) apply. In the above quoted magnetic thermally insulated arc experiment for example, the arc radius was of the order cm, and it was found that for an arc length of 40 cm and at which a temperature of $\sim 10^5$ °K was reached, the end losses predominated the radial heat conduction losses. The authors of this experiment therefore concluded that if the arc would have been made ~ 10 meters long, a temperature of $\sim 10^6$ °K could have been reached.

The losses from the two ends of the arc are

$$\phi = 2 \times (1/2)\,\varrho v^3 \pi r^2, \tag{17}$$

where ϱ is the average density and v the average axial velocity of the arc plasma. Putting $v = \sqrt{kT/m_i}$, and $\varrho = nm_i = m_i p/kT$, we have

$$\phi = p(kT/m_i)^{1/2} \pi r^2. \tag{18}$$

If we neglect the radial heat conduction losses, we can equate (18) with the input power EIl for the entire arc:

$$EI = p(kT/m_i)^{1/2} \pi r^2/l. \tag{19}$$

For arcs which are limited by their end losses and which have the same input power, the following similarity law then holds:

$$Tr^4 = \text{const.} \tag{20}$$

From this relation it follows that a two-fold reduction in the arc radius would result in a ~ 10 fold increase of the arc temperature. Applied to the above quoted experiment, where the arc radius was of the order ~ 1 cm, and a temperature of ~ 10^5 °K was reached at an arc length of ~ 40 cm, a reduction of the arc radius down to 0.5 cm could there have raised the arc temperature to ~ 10^6 °K. This reasoning underlines the importance to reduce the arc radius for the attainment of the highest possible temperatures.

4. Arc in Supersonic Vortex with Magnetic Thermal Insulation

As we had seen in the previous chapter, for magnetic thermally insulated arcs the ratio of the arc radius to the arc length is normally so large, that these arcs are limited by their end losses. Much higher arc temperatures could be expected, if the arc radius could be reduced to such an extent, that for arcs with a length of a few cm these end losses would become insignificant. Arcs having this property could then take full advantage of the magnetic thermal insulation effect. We will now show how this goal might be achieved by combining the Gerdien arc technique with magnetic thermal insulation.

For the working of the Gerdien arc concept it is important that the vortex has a sharp boundary, establishing a vacuum in its core, because only then is it possible to confine the arc inside the core. The density of the arc is normally much smaller than the density of the vortex, resulting in strong buoyancy forces acting on the arc. There is no problem to make a vortex with a sharp boundary if water or some other liquid is used. In applying a magnetic field to a water vortex Gerdien arc with the goal to reach much higher temperatures, not much would be gained because there the radiation losses from the emission of oxygen would become prohibitive above 10^5 °K. Superficially it might appear, that the only way to reduce these radiation losses is to make a vortex from liquid hydrogen. Quite apart from the considerable technical difficulties this idea fails at the envisioned high arc temperatures, because the liquid hydrogen vortex would there rapidly vaporize, with the result that the hollow core is destroyed.

A way out proposed here, is to make a hydrogen gas vortex. However, simply replacing a liquid vortex by swirling hydrogen gas will in general not produce a vortex core with a sharp boundary. This is only possible if the inner part of the vortex becomes supersonic, because only then has the vortex a vacuum core, as it is also the case for a liquid vortex.

The azimuthal velocity of an irrotational flow gas vortex with an inner core radius r_0 is given by [11, 12]

$$v = v_0 (r_0/r), \tag{21}$$

where [13]

$$v_0 = c_0 \sqrt{2/(\gamma - 1)} \tag{22}$$

and

$$c_0^2 = \gamma p_0/\varrho_0 = \gamma R T_0/\mu . \tag{23}$$

The values p_0, ϱ_0, T_0 (pressure, density, temperature) are taken at $r = \infty$. γ is the specific heat ratio, μ the molecular weight and R the gas constant. From the expression for the velocity of sound in the flowing medium

$$c^2 = c_0^2 - ((\gamma - 1)/2) v^2 \tag{24}$$

one obtains for the Mach number M:

$$\begin{aligned} M^2 = \frac{v^2}{c^2} &= \frac{v^2}{c_0^2 - ((\gamma - 1)/2) v^2} \\ &= [2/(\gamma - 1)/((r/r_0)^2 - 1)] . \end{aligned} \tag{25}$$

$M = 1$ is reached at the radius $r_c > r_0$ given by

$$\frac{r_c}{r_0} = \sqrt{\frac{\gamma + 1}{\gamma - 1}} . \tag{26}$$

For $r_0 < r < r_c$ the vortex is therefore supersonic, reaching $M = \infty$ at $r = r_0$, and for $r > r_c$ the vortex is subsonic.

Finally, the pressure and density of the vortex are

$$\begin{aligned} p/p_0 &= [1 - ((\gamma - 1)/2) v^2/c_0^2]^{\gamma/\gamma - 1} \\ &= [1 - (r_0/r)^2]^{\gamma/\gamma - 1} \end{aligned} \tag{27}$$

$$\begin{aligned} \varrho/\varrho_0 &= [1 - ((\gamma - 1)/2) v^2/c^2]^{1/\gamma - 1} \\ &= [1 - (r_0/r)^2]^{1/\gamma - 1} . \end{aligned} \tag{28}$$

The dependence of $v(r)$, $p(r)$ and $\varrho(r)$ are shown in Figure 1. It can be seen, that the density in the vortex as a function of the radial distance from the axis rises rapidly. As a result, large buoyancy forces are going to act on an arc burning through the core.

The heat transfer from the burning arc into the hydrogen gas vortex is going to raise the gas temperature and thereby will lead to a reduction in the Mach number. If the temperature rise is too large, the vortex core can become subsonic. If this happens, the gas

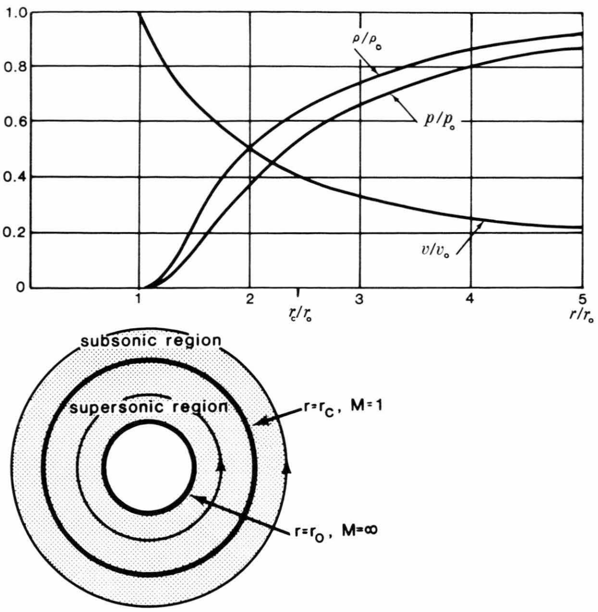

Fig. 1. Supersonic vortex; v, p and ϱ in dependence of r/r_0. $r = r_c$ radial distance below which flow becomes supersonic.

pressure in the center of the vortex will rise and destroy the vacuum in the core. And as a result the hydrogen gas will strongly mix with the arc plasma. To prevent this from happening, the flow of the hydrogen gas must be chosen sufficiently large to ensure that enough heat is continuously removed by the axial gas flow. The vortex can then be kept cool and supersonic.

One has (c_p specific heat at constant pressure)

$$v_0^2/2 = c_p T_0 \tag{29}$$

and hence

$$\frac{v_0^2}{2}\frac{\mathrm{d}m}{\mathrm{d}t} = c_p T_0 \frac{\mathrm{d}m}{\mathrm{d}t}, \tag{30}$$

where $\mathrm{d}m/\mathrm{d}t$ is hydrogen mass flow rate. The heat transfer from the arc and which has the rate $I E l$, leads to a temperature rise ΔT given by

$$c_p \Delta T \frac{\mathrm{d}m}{\mathrm{d}t} = I E l. \tag{31}$$

One therefore has

$$\frac{\Delta T}{T_0} = \frac{2 I E l}{v_0^2 \,\mathrm{d}m/\mathrm{d}t}. \tag{32}$$

A rise in temperature by ΔT will imply a rise in the square of the sound velocity by Δc^2. To keep $M^2 > 1$ implies according to (25), that

$$c_0^2 - \Delta c^2 - ((\gamma-1)/2)\, v^2 < v^2. \tag{33}$$

Putting $v^2 = v_0^2 = 2 c_0^2/\gamma - 1)$ this means that

$$\Delta c^2 < v^2 \tag{34}$$

therefore

$$\Delta T < T_0 \tag{34a}$$

and hence

$$\mathrm{d}m/\mathrm{d}t > 2 I E l/v_0^2 = I E l/c_p T_0. \tag{35}$$

From the Elenbaas-Heller equation (8) it follows that the radial temperature distribution in the hydrogen gas has a logarithmic dependence. The inner part of the vortex is therefore not getting much hotter than the outer part. Would it be otherwise, the inner part of the vortex could become subsonic even if inequality (35) would be satisfied.

If the vortex core and hence the arc radius can be made sufficiently small, the end losses become insignificant above a certain arc length. The condition for this to happen is simply

$$\phi/l \ll I E. \tag{36}$$

If this condition is satisfied the arc losses are determined (apart from the radiation losses which for hydrogen are small by comparison) by the radial heat conduction losses alone, and the integrated Elenbaas-Heller equation (16) does apply. (The similarity law (20) and which was derived for end-loss dominated arcs, of course, does then not more apply). The arc is now entirely determined by the integrated Elenbaas-Heller equation and Ohm's law, Equation (4).

From condition (36) follows a condition for the arc length if we insert the expression of $I E$ from the Elenbaas-Heller equation (16) with magnetic thermal insulation:

$$l > 2.2 \times 10^{-10} (kT/m_i)^{1/2} p^{-0.4} H^{1.4} \pi r_0^2 \ln(r_a/r_0) \tag{37}$$

Another constraint on the arc length is that $l > \lambda$, where λ is the mean free path in the arc plasma, because Ohmic heating will be effective only as long as this condition is satisfied. At $T \sim 10^6$ °K the scattering cross section and which determines the mean free path is $\sigma_s \sim 10^{-16}$ cm^2. It is therefore required that $l > 1/n\sigma_s \simeq 10^{16}/n$.

Applying a strong axial magnetic field on the arc entrapped in the center of the vortex, there will be an induced azimuthal current caused by the thermomagnetic Nernst effect, with the current density (in cgs units) given by

$$j = \frac{3nkc}{2H^2} \boldsymbol{H} \times \nabla T. \tag{38}$$

The occurrence of this current will result in a change of the plasma equilibrium, governed by the equation:

$$\nabla p = \frac{1}{c} \boldsymbol{j} \times \boldsymbol{H} = \frac{3}{2} \frac{nk}{H^2} (\boldsymbol{H} \times \nabla T) \times \boldsymbol{H}. \tag{39}$$

For a hydrogen plasma $p = 2nkT$, (where n is here the number density of hydrogen ions), one has

$$\nabla p = 2nk\nabla T + 2kT\nabla n = (3/2)\, nk\nabla T \tag{40}$$

which upon integration yields [9]

$$T n^4 = \text{const}, \tag{41}$$

instead of $Tn = \text{const}$, as it would be the case without the Nernst effect. Because the arc temperature is a decreasing function of r, the plasma density n increases much more slowly than without the Nernst effect. This means, a magnetic field not only reduces the radial heat conduction losses, but also the plasma density towards the inner core surface of the confining gas vortex. As a result one has a more uniform plasma density across the arc. In case the core radius is smaller than the mean free path in the plasma, the reduction in the density towards the surface of the arc reduces the friction in between the arc plasma and the vortex.

The Nernst effect also leads to a reduction of the radiation losses. For temperatures above 10^5 °K they are dominated by free-free bremsstrahlung, and are [14]

$$\mathscr{E}_r = 1.42 \times 10^{-27} n^2 \sqrt{T}. \tag{42}$$

Without the Nernst effect, where $nT = \text{const}$, these losses rise radially with decreasing temperature as $T^{-3/2}$, but with the Nernst effect where $T n^4 = \text{const}$ they are constant across the entire arc.

In keeping down the radiation losses one other serious problem would be if impurities from the vaporization of the electrodes could enter the arc. In the Gerdien arc concept this problem is completely eliminated due to the axial plasma flow towards both electrodes, with fresh hydrogen from the erosion of the vortex continuously replacing these losses. The arc therefore cleans itself against impurities. Impurities could still be introduced in a controlled way by adding them to the hydrogen injected from outside to sustain the supersonic vortex.

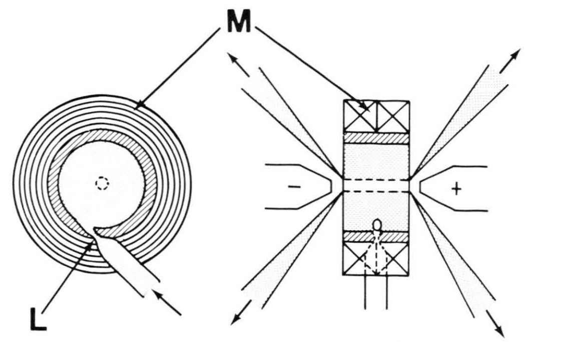

Fig. 2. Establishing a supersonic gas vortex by the injection through a Laval nozzle L into arc discharge chamber.

Figure 2 shows how the hydrogen gas could be radially injected into the arc chamber through a Laval nozzle. Because the flow emerging from a Laval nozzle is supersonic, it may in this way be possible to generate a largely supersonic vortex, with a subsonic outer region only in the boundary layer of the wall of the arc chamber.

Combining (35) with the expression for IE given by (16) and the inequality for l given by (37), one sees that dm/dt scales as the pressure. This means, that for a chosen set of arc parameters the hydrogen consumption can be reduced. Inserting (19) into (35) one finds

$$dm/dt > p\pi r_0^2 (kT/m_i)^{1/2}/c_p T_0. \tag{43}$$

As mentioned above, the idea to stabilize an arc in a rotating gas column as such is not new and goes back to Schoenherr [2]. It has in the past also been used by Pfender and Betz [15], and by Braams [16]. However, it appears that these authors have overlooked the crucial importance of a supersonic vortex. In a subsonic vortex there is no sharply defined empty vortex core, resulting in a large mixing of the arc plasma with the gas vortex. The arc radius is there diffuse, and the arc cannot be confined into a narrow channel. The much larger buoyancy force in a supersonic vortex not only enhances the overall stability of the arc plasma, but makes possible its stable confinement inside the narrow vortex core.

5. An Example

We assume an arc entrapped in a vortex with a core radius $r_0 = 0.2$ cm. Its temperature shall be 10^6 °K, and it shall have a number density $n = 10^{16}$ cm^{-3}. At a temperature of 10^6 °K it would then produce a pressure of 1 atmosphere. The strength of the applied magnetic field shall be $H = 20$ kG as it can be produced with ordinary electromagnets.

Putting $\ln(r_a/r) \sim 3$, we find from (16) that in order to balance the radial heat conduction losses one would have to put $EI \simeq 40$ kW/cm. The value for $\omega_i \tau_i$, (12), turns out to be slightly larger than one. According to the remark following (14) this means that the arc temperature is approximately equal to the temperature of the maximum ion conductivity. The radiation losses are computed from (42), and one finds $\pi r_0^2 \varepsilon_r \simeq 2$ W/cm. They are therefore very small compared to the conduction losses. From the condition (37) one computes that the arc length must be $l > 3$ cm. Therefore, an arc ~ 10 cm long should already approximately satisfy this condition. For $n = 10^{16}\ \mathrm{cm}^{-3}$ the mean free path is $\lambda \sim 1$ cm and for $l = 10$ cm, the condition $\lambda \ll l$ is therefore satisfied. The input power for this arc would be $IEl = 400$ kW. From (4) one then obtains that $E \sim 20$ Volt/cm, hence $V = El = 200$ Volt. With $IEl = 4 \times 10^5$ Watt it follows that $I = 2 \times 10^3$ Ampere. Finally, from (35), putting $2IE \simeq 10^{13}$ erg/sec, it follows that $\mathrm{d}m/\mathrm{d}t > 100$ g/sec. Since according to (43) $\mathrm{d}m/\mathrm{d}t$ scales in proportion with the pressure, this rather large hydrogen consumption could be somewhat reduced by reducing the pressure. However, it is not possible to reduce the pressure below 0.1 atmospheres without violating the condition that $\lambda \ll l$, for the given example. At $p = 0.1$ atmospheres one would have $\mathrm{d}m/dt > 10$ g/sec. In lowering the pressure tenfold, the input power to drive the arc would likewise be reduced ten-fold. At an unchanged arc resistance the current would be reduced to ~ 200 Ampere. A further reduction in $\mathrm{d}m/dt$ is possible if one can reduce r_0 to even smaller values. The success in reducing r_0 to very small values will depend on how accurate one can control the flow in the vortex. At temperatures below $\sim 10^6$ °K the hydrogen consumption will, of course, also be less.

These numbers indicate, that arc temperatures well in excess of 10^5 °K and perhaps up to 10^6 °K might be achieved with the proposed technique for arcs having a rather small size.

6. Two Applications

First we would like to discuss the use of the arc as an intense radiation source. The emitted bremsstrahlungsradiation by free-free transitions, given by (42), was for the chosen example a modest ~ 2 W/cm. This radiation can be easily increased by adding impurities to the hydrogen, whereby through a proper choice of the impurities, line emission at high intensities within a narrow frequency band can be achieved.

If the arc would burn as a black body source, the radiation emitted from both of its ends would be (at the parameters used in our example, σ Stefan-Boltzmann constant):

$$I_s = 2\pi r_0^2 \sigma T^4 . \tag{44}$$

The maximum radiation output of the arc can, of course not exceed the electric power input $P = EIl = 400$ kW. The black body radiation limit can for this reason alone never be reached.

The emission of "grey" body radiation in relation to black body radiation is

$$I_g \simeq I_s(1 - e^{-l/\lambda_p}), \tag{45}$$

where λ_p is the photon mean free path. If $l \ll \lambda_p$ we can expand the r.h.s. of (45) and have

$$I_g \simeq I_s l/\lambda_p . \tag{46}$$

For hydrogen, I_g must be equal to $\pi r_0^2 l \varepsilon_r$. Equating this with (46), we obtain an expression for λ_p, valid for hydrogen*:

$$\lambda_p \simeq 10^{23}\, T^{2.5}/n^2 . \tag{47}$$

For $n \simeq 10^{16}\ \mathrm{cm}^{-3}$, and $T \sim 10^6$ °K it follows that $l \ll \lambda_p$ is there very well satisfied and the hydrogen arc plasma is for all practical purposes transparent.

By adding impurities to the arc plasma it is possible to increase the radiation output substantially over that of bremsstrahlung in hydrogen. Obviously however, the power of the emitted radiation cannot exceed the electric input power P. This condition therefore implies that

$$I_g < P \tag{48}$$

In our example $P = 4 \times 10^5$ Watt. According to (46) one has

$$\lambda_p/l = I_s/I_g \simeq 3 \times 10^6 \tag{49}$$

* In astrophysics one uses instead of λ_p the opacity coefficient $\varkappa$ related to λ_p by $\lambda_p = 1/\varrho\varkappa$, which is given in the form [17] $\varkappa = \varkappa_0 \varrho T^{-3.5}$, where ϱ is the plasma density and where $\varkappa_0$ depends on the chemical composition of the plasma.

which shows, that even here the arc plasma is still very much transparent. From $\lambda_p = 1/n\,\sigma_p$, we can compute the effective photon absorption cross section which is (for $n \simeq 10^{16}\ \mathrm{cm}^{-3}$) $\sigma_p \simeq 3 \times 10^{-22}\ \mathrm{cm}^2$. For heavier atoms $\sigma_p \simeq 10^{-18}\ \mathrm{cm}^2$ is a typical value, which means that by an impurity fraction of $\sim 10^{-4}$ the maximum radiative power could be reached. Of course, even if $I_g \simeq P$, only the fraction $r/l \simeq 2 \times 10^{-2}$ can go into the direction of the two ends of the arc. This maximum power is ~ 10 kW.

We may compare this number with a proposed synchrotron source. There the maximum photon energy flux at a photon energy of ~ 100 eV and which corresponds to our $\sim 10^6\ ^\circ$K, is only ~ 100 Watt. Therefore, even if we remain about a factor 10 below our maximum value of ~ 10 kW, we would be still better by an order of magnitude than the quoted synchrotron source.

A second interesting application of the hot arc plasma is for the generation of ultrafast supersonic jets [18]. For this purpose one end (or both) of the arc have to be connected to a Laval nozzle. If the arc temperature is T, the maximum exhaust velocity of a hydrogen jet is given by (R is the gas constant):

$$v = \sqrt{5\,R\,T}. \tag{50}$$

For $T = 10^5\ ^\circ$K we have $v \simeq 60$ km/sec and for $T = 10^6\ ^\circ$K, $v \simeq 200$ km/sec. It therefore seems quite well possible to reach with a $\sim 10^6\ ^\circ$K arc, jet velocities of ~ 100 km/sec.

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